

# Single-spin Azimuthal Asymmetries in the “Reduced Twist-3 Approximation”

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## Abstract

We consider the single-spin azimuthal asymmetries recently measured at the HERMES experiment for charged pions produced in semi-inclusive deep inelastic scattering of leptons off longitudinally polarized protons. Guided by the experimental results and assuming a vanishing twist-2 *transverse* quark spin distribution in the *longitudinally* polarized nucleon, denoted as “reduced twist-3 approximation”, a self-consistent description of the observed single-spin asymmetries is obtained. In addition, predictions are given for the  $z$  dependence of the single target-spin asymmetry.

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Semi-inclusive deep inelastic scattering (SIDIS) of leptons off a polarized nucleon target is a rich source of information on the spin structure of the nucleon and on parton fragmentation. In particular, measurements of azimuthal asymmetries in SIDIS allow the further investigation of the quark and gluon structure of the polarized nucleon. The HERMES collaboration has recently reported on the measurement of single target-spin asymmetries in the distribution of the azimuthal angle  $\phi$  relative to the lepton scattering plane, in semi-inclusive charged pion production on a longitudinally polarized hydrogen target [1]. The  $\sin\phi$  moment of this distribution was found to be significant for  $\pi^+$ -production. For  $\pi^-$  it was found to be consistent with zero within present experimental uncertainties, as it was the case for the  $\sin 2\phi$  moments of both  $\pi^+$  and  $\pi^-$ . Single-spin asymmetries vanish in models in which hadrons consist of non-interacting collinear partons (quarks and gluons), i.e. they are forbidden in the simplest version of the parton model and perturbative QCD. Non-vanishing and non-identical intrinsic transverse momentum distributions for oppositely polarized partons play an important role in most explanations of such non-zero single-spin asymmetries; they are interpreted as the effects of “naive time-reversal-odd” (T-odd) fragmentation functions [2-6], arising from non-perturbative hadronic final-state interactions. In Refs. [7, 8] these asymmetries were evaluated and it was shown that a good agreement with the HERMES data can be achieved by using only twist-2 distribution and fragmentation functions.

In this letter the single target-spin  $\sin\phi_h$  and  $\sin 2\phi_h$  azimuthal asymmetries are investigated in the light of the recent HERMES results [1]. It will be shown that these results may be interpreted towards a vanishing twist-2 quark transverse spin distribution in the longitudinally polarized nucleon <sup>2</sup>. Under this assumption, which will be called hereafter “reduced twist-3 approximation”, the sub-leading order in  $1/Q$  single target-spin  $\sin\phi_h$  asymmetry reduces to the twist-2 level and is interpreted as the effect of the convolution of the transversity distribution and the T-odd fragmentation function. In this situation, also measurements with a longitudinally polarized target at HERMES may be used to extract the transversity distribution in a way similar to that proposed in Ref. [11] for a transversely polarized target, once enough statistics will be collected.

The  $\sin\phi_h$  and  $\sin 2\phi_h$  moments of experimentally observable single target-spin asymmetries in the SIDIS cross-section can be related to the parton distribution and fragmentation functions involved in the parton level description of the underlying process [3, 4]. Their anticipated dependence on  $p_T$  ( $k_T$ ), the intrinsic transverse momentum of the initial (final) parton, reflects into the distribution of  $P_{hT}$ , the transverse momentum of the semi-inclusively measured hadron. The moments are defined as appropriately weighted integrals over this observable, of the cross section asymmetry:

$$\left\langle \frac{|P_{hT}|}{M_h} \sin\phi_h \right\rangle \equiv \frac{\int d^2 P_{hT} \frac{|P_{hT}|}{M_h} \sin\phi_h (d\sigma^+ - d\sigma^-)}{\int d^2 P_{hT} (d\sigma^+ + d\sigma^-)}, \quad (1)$$

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<sup>2</sup>After this work has been completed we became aware of Refs. [9, 10] where this possibility has also been considered.

$$\langle \frac{|P_{hT}|^2}{MM_h} \sin 2\phi_h \rangle \equiv \frac{\int d^2 P_{hT} \frac{|P_{hT}|^2}{MM_h} \sin 2\phi_h (d\sigma^+ - d\sigma^-)}{\int d^2 P_{hT} (d\sigma^+ + d\sigma^-)}. \quad (2)$$

Here  $+$  ( $-$ ) denote the antiparallel (parallel) longitudinal polarization of the target and  $M$  ( $M_h$ ) is the mass of the target (final hadron). For both polarized and unpolarized leptons these asymmetries are given by [3, 4, 12]<sup>3</sup>

$$\langle \frac{|P_{hT}|}{M_h} \sin \phi_h \rangle(x, y, z) = \frac{1}{I_0(x, y, z)} [I_{1L}(x, y, z) + I_{1T}(x, y, z)], \quad (3)$$

$$\langle \frac{|P_{hT}|^2}{MM_h} \sin 2\phi_h \rangle(x, y, z) = \frac{8}{I_0(x, y, z)} S_L (1 - y) h_{1L}^{\perp(1)}(x) z^2 H_1^{\perp(1)}(z), \quad (4)$$

where

$$\begin{aligned} I_0(x, y, z) &= (1 + (1 + y)^2) f_1(x) D_1(z), \\ I_{1L}(x, y, z) &= 4S_L \frac{M}{Q} (2 - y) \sqrt{1 - y} [x h_L(x) z H_1^{\perp(1)}(z) - h_{1L}^{\perp(1)}(x) \tilde{H}(z)], \\ I_{1T}(x, y, z) &= 2S_{Tx} (1 - y) h_1(x) z H_1^{\perp(1)}(z). \end{aligned} \quad (5)$$

With  $k_1$  ( $k_2$ ) being the 4-momentum of the incoming (outgoing) charged lepton,  $Q^2 = -q^2$  where  $q = k_1 - k_2$  is the 4-momentum of the virtual photon.  $P$  ( $P_h$ ) is the momentum of the target (final hadron),  $x = Q^2/2(Pq)$ ,  $y = (Pq)/(Pk_1)$ ,  $z = (PP_h)/(Pq)$ ,  $k_{1T}$  the incoming lepton transverse momentum with respect to the virtual photon momentum direction, and  $\phi_h$  is the azimuthal angle between  $P_{hT}$  and  $k_{1T}$  around the virtual photon direction. Note that the azimuthal angle of the transverse (with respect to the virtual photon) component of the target polarization,  $\phi_S$ , is equal to 0 ( $\pi$ ) for the target polarized parallel (anti-parallel) to the beam [12]. The components of the longitudinal and transverse target polarization in the virtual photon frame are denoted by  $S_L$  and  $S_{Tx}$ , respectively. Twist-2 distribution and fragmentation functions have a subscript ‘1’:  $f_1(x)$  and  $D_1(z)$  are the usual unpolarized distribution and fragmentation functions, while  $h_{1L}^{\perp(1)}(x)$  and  $h_1(x)$  describe the quark transverse spin distribution in longitudinally and transversely polarized nucleons, respectively. The twist-3 distribution function in the longitudinally polarized nucleon is denoted by  $h_L(x)$  [14]. The spin dependent fragmentation function  $H_1^{\perp(1)}(z)$ , describing transversely polarized quark fragmentation (Collins effect [2]), can be interpreted as the production probability of an unpolarized hadron from a transversely polarized quark [15]. The fragmentation function  $\tilde{H}(z)$  is the interaction-dependent part of the twist-3 fragmentation function:  $H(z) = -2zH_1^{\perp(1)}(z) + \tilde{H}(z)$ . The functions with superscript (1) denote  $p_T^2$ - and  $k_T^2$ -moments, respectively:

$$h_{1L}^{\perp(1)}(x) \equiv \int d^2 p_T \left( \frac{p_T^2}{2M^2} \right) h_{1L}^{\perp}(x, p_T^2), \quad (7)$$

$$H_1^{\perp(1)}(z) \equiv z^2 \int d^2 k_T \left( \frac{k_T^2}{2M_h^2} \right) H_1^{\perp}(z, z^2 k_T^2). \quad (8)$$

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<sup>3</sup>We omit the current quark mass dependent terms.

The function  $h_L(x)$  can be split into a twist-2 part,  $h_{1L}^{\perp(1)}(x)$ , and an interaction-dependent part,  $\tilde{h}_L(x)$  [14, 16]:

$$h_L(x) = -2 \frac{h_{1L}^{\perp(1)}(x)}{x} + \tilde{h}_L(x). \quad (9)$$

As it was shown in Refs. [16, 4] this relation can be rewritten as

$$h_L(x) = h_1(x) - \frac{d}{dx} h_{1L}^{\perp(1)}(x). \quad (10)$$

The weighted single target-spin asymmetries defined above are related to the ones measured by HERMES [1] through the following relations:

$$A_{UL}^{\sin \phi_h} \approx \frac{2M_h}{\langle P_{hT} \rangle} \langle \frac{|P_{hT}|}{M_h} \sin \phi_h \rangle, \quad (11)$$

$$A_{UL}^{\sin 2\phi_h} \approx \frac{2MM_h}{\langle P_{hT}^2 \rangle} \langle \frac{|P_{hT}|^2}{MM_h} \sin 2\phi_h \rangle, \quad (12)$$

where the subscripts  $U$  and  $L$  indicate unpolarized beam and longitudinally polarized target, respectively.

When combining the HERMES experimental results of a significant target-spin  $\sin \phi_h$  asymmetry for  $\pi^+$  and of a vanishing  $\sin 2\phi_h$  asymmetry with the preliminary evidence from  $Z^0 \rightarrow 2$ -jet decay on a non-zero T-odd transversely polarized quark fragmentation function [17], it follows immediately from Eq.(4) that  $h_{1L}^{\perp(1)}(x)$ , the twist-2 transverse quark spin distribution in a longitudinally polarized nucleon, should vanish. Consequently, from Eqs. (9, 10) follows that

$$h_L(x) = \tilde{h}_L(x) = h_1(x). \quad (13)$$

In this situation the single target-spin  $\sin \phi_h$  asymmetry given by Eq.(3) reduces to the twist-2 level (“reduced twist-3 approximation”). The fact that  $h_{1L}^{\perp(1)}(x)$  (see Eq.7) vanishes may be interpreted as follows: the distribution function  $h_{1L}^{\perp}(x, p_T^2)$ , which is non-zero itself, vanishes at any  $x$  when it is averaged over the intrinsic transverse momentum of the initial parton,  $p_T$ . As a matter of fact, in a longitudinally polarized nucleon partons polarized transversely at large  $p_T$  may indeed have a polarization opposite to that at smaller  $p_T$ , at any  $x$ .

It is important to mention that the “reduced twist-3 approximation” does not require  $\tilde{H}(z) = 0$ , which otherwise would lead to the inconsistency that  $H_1^{\perp(1)}(z)$  would be required to vanish [4, 18].

For the numerical calculations the non-relativistic approximation  $h_1(x) = g_1(x)$  is taken as lower limit <sup>4</sup>, and  $h_1(x) = (f_1(x) + g_1(x))/2$  as an upper limit [23]. For the

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<sup>4</sup>For non-relativistic quarks  $h_1(x) = g_1(x)$ . Several models suggest that  $h_1(x)$  has the same order of magnitude as  $g_1$  [13, 19, 20]. The evolution properties of  $h_1$  and  $g_1$ , however, are very different [21, 22]. At the  $Q^2$  values of the HERMES measurement the assumption  $h_1 = g_1$  is fulfilled at large, i.e. valence-like,  $x$  values, while large differences occur at lower  $x$  [11].

sake of simplicity,  $Q^2$ -independent parameterizations were chosen for the distribution functions  $f_1(x)$  and  $g_1(x)$  [24].

To calculate the T-odd fragmentation function  $H_1^{\perp(1)}(z)$ , the Collins parameterization [2] for the analyzing power of transversely polarized quark fragmentation was adopted:

$$A_C(z, k_T) \equiv \frac{|k_T|}{M_h} \frac{H_1^\perp(z, k_T^2)}{D_1(z, k_T^2)} = \frac{M_C |k_T|}{M_C^2 + k_T^2} \quad (14)$$

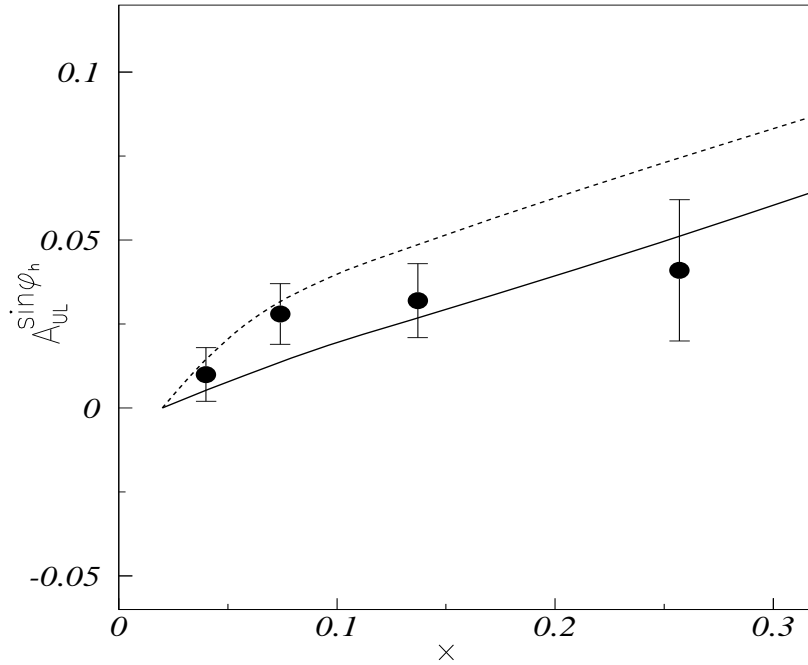


Figure 1: The single target-spin asymmetry  $A_{UL}^{\sin \phi_h}$  for  $\pi^+$  production as a function of Bjorken  $x$ , evaluated using  $M_C = 0.28$  GeV in Eq.(14). The solid line corresponds to  $h_1 = g_1$ , the dashed one to  $h_1 = (f_1 + g_1)/2$ . Data are from Ref. [1].

For the distribution of the final parton's intrinsic transverse momentum,  $k_T$ , in the unpolarized fragmentation function  $D_1(z, k_T^2)$  a Gaussian parameterization was used [25] with  $\langle z^2 k_T^2 \rangle = b^2$  (in the numerical calculations  $b = 0.36$  GeV was taken [26]). For  $D_1^{\pi^+}(z)$  the parameterization from Ref. [27] was adopted. In Eq.(14)  $M_C$  is a typical hadronic mass whose value may range from  $m_\pi$  to  $M_p$ . Using  $M_C = 2m_\pi$  for the analyzing power of Eq.(14) results in

$$\frac{\int_{z_0=0.1}^1 dz H_1^\perp(z)}{\int_{z_0=0.1}^1 dz D_1(z)} = 0.062, \quad (15)$$

which is in good agreement with the experimental result  $0.063 \pm 0.017$  given for this ratio in Ref. [17]. Here  $H_1^\perp(z)$  is the unweighted polarized fragmentation function, defined as:

$$H_1^\perp(z) \equiv z^2 \int d^2 k_T H_1^\perp(z, z^2 k_T^2). \quad (16)$$

It is worth mentioning that the ratio in Eq.(15) is rather sensitive to the lower limit of integration,  $z_0$  [28]. By using  $z_0 = 0.01$ , the ratio reduces to 0.03; choosing a value of  $z_0$  equal to 0.2 (0.3), the ratio increases to about 0.1 (0.12). This behavior is mainly due to the fact that the fragmentation function  $D_1(z)$  diverges at small values of  $z$ .

In Fig. 1, the asymmetry  $A_{UL}^{\sin\phi_h}(x)$  of Eq.(11) for  $\pi^+$  production on a proton target is presented as a function of  $x$ -Bjorken and compared to HERMES data [1], which correspond to  $1 \text{ GeV}^2 \leq Q^2 \leq 15 \text{ GeV}^2$ ,  $4 \text{ GeV} \leq E_\pi \leq 13.5 \text{ GeV}$ ,  $0.02 \leq x \leq 0.4$ ,  $0.2 \leq z \leq 0.7$ , and  $0.2 \leq y \leq 0.8$ . The two theoretical curves are calculated by integrating over the same kinematic ranges taking  $\langle P_{hT} \rangle = 0.365 \text{ GeV}$  as input. The latter value is obtained in this kinematic region assuming a Gaussian parameterization of the distribution and fragmentation functions with  $\langle p_T^2 \rangle = (0.44)^2 \text{ GeV}^2$  [26]. From Fig. 1 it can be concluded that there is good agreement between the calculation in this letter and the HERMES data. Note that the ‘kinematic’ contribution to  $A_{UL}^{\sin\phi_h}(x)$ , coming from the transverse component of the target polarization (with respect to the virtual photon direction) and given by  $I_{1T}$  (Eq.(6)), amounts to only 25%.

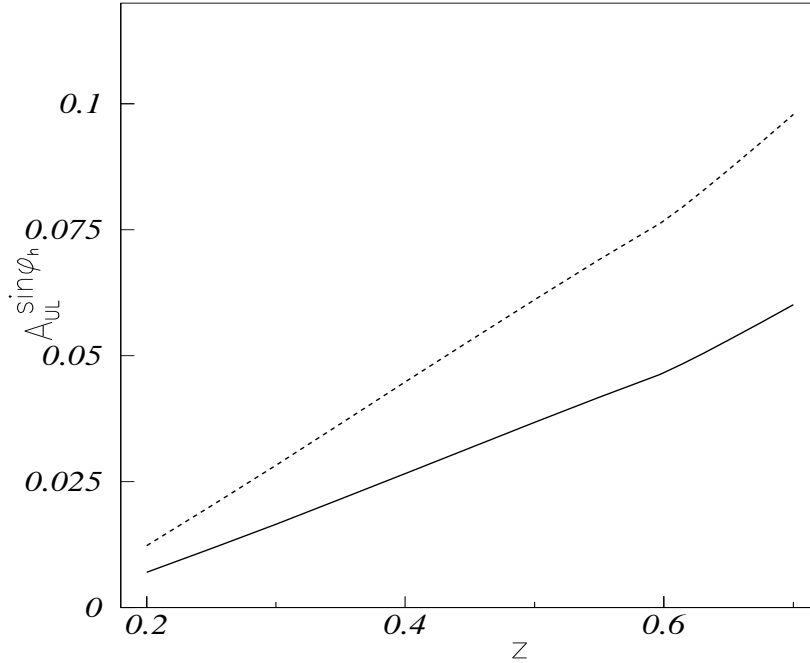


Figure 2: The single target-spin asymmetry  $A_{UL}^{\sin\phi_h}$  for  $\pi^+$  production as a function of  $z$  evaluated using  $M_C = 0.28 \text{ GeV}$ . The solid line corresponds to  $h_1 = g_1$ , the dashed one to  $h_1 = (f_1 + g_1)/2$ .

The  $z$  dependence of the asymmetry  $A_{UL}^{\sin\phi_h}$  for  $\pi^+$  production is shown in Fig. 2, where the two curves correspond to two limits for  $h_1(x)$ , as introduced above. No data are available yet to constrain the calculations.

In conclusion, the recently observed single-spin azimuthal asymmetries in semi-inclusive deep inelastic lepton scattering off a longitudinally polarized proton target at HERMES are interpreted on the basis of the so-called “reduced twist-3 approximation”, that is assuming a vanishing twist-2 transverse quark spin distribution in the longitudinally polarized nucleon. This leads to a self-consistent description of the observed single-spin asymmetries. In this approach the target-spin  $\sin\phi_h$  asymmetry is interpreted as the effect of the convolution of the transversity distribution,  $h_1(x)$ , and a T-odd fragmentation function,  $H_1^{\perp(1)}(z)$ , and may allow to probe transverse spin observables in a longitudinally polarized nucleon.

In addition, predictions are given for the  $z$  dependence of the single target-spin  $\sin\phi_h$  asymmetry, for which experimental data are not yet published.

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